

Chapter 5 Upthrust in Fluids, Archimedes' Principle and Floatation

Exercise (A)

1 . What do you understand by the term upthrust of a fluid? Describe an experiment to show its existence.

Solution

When a body is partially or wholly immersed in a liquid, an upward force acts on it. This upward force is known as an upthrust.

Upthrust can be demonstrated by the following experiment:

Take an empty can and close its mouth with an airtight stopper. Put it in a tub filled with water. It floats with a large part of it above the surface of water and only a small part of it below the surface of water. Push the can into the water. You can feel an upward force and you find it difficult to push the can further into water. It is noticed that as the can is pushed more and more into the water, more and more force is needed to push the can further into water, until it is completely immersed. When the can is fully inside the water, a definite force is still needed to keep it at rest in that position. Again, if the can is released in this position, it is noticed that the can bounces back to the surface and starts floating again.

2. In what direction and at what point does the buoyant force on a body due to a liquid act?

Solution 2

Buoyant force on a body due to a liquid acts upwards at the centre of buoyancy.

3.What is meant by the term buoyancy?

Solution

The property of a liquid to exert an upward force on a body immersed in it is called buoyancy.

4. Define upthrust and state its S.I. unit.

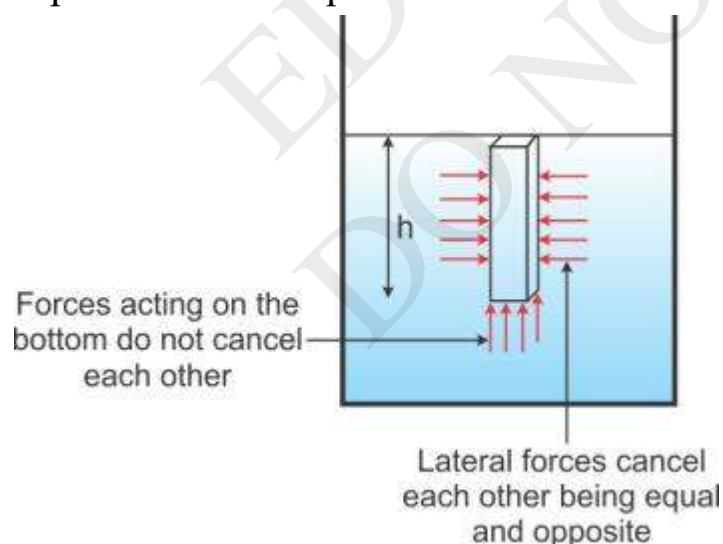
Solution

The upward force exerted on a body by the fluid in which it is submerged is called the upthrust. Its S.I. unit is 'newton'.

5.What is the cause of upthrust? At which point can it be considered to act?

Solution

A liquid contained in a vessel exerts pressure at all points and in all directions. The pressure at a point in a liquid is the same in all directions - upwards, downwards and sideways. It increases with the depth inside the liquid.



When a body is immersed in a liquid, the thrusts acting on the side walls of the body are neutralized as they are equal in magnitude and opposite in direction. However, the magnitudes of pressure on the

upper and lower faces are not equal. The difference in pressure on the upper and lower faces cause a net upward force ($= \text{pressure} \times \text{area}$) or upthrust on the body.

It acts at the centre of buoyancy.

6. Why is a force needed to keep a block of wood inside water?

Solution

Upthrust due to water on block when fully submerged is more than its weight. Density of water is more than the density of cork; hence, upthrust due to water on the block of cork when fully submerged in water is more than its weight.

7.A piece of wood if left under water comes to the surface. Explain the reason.

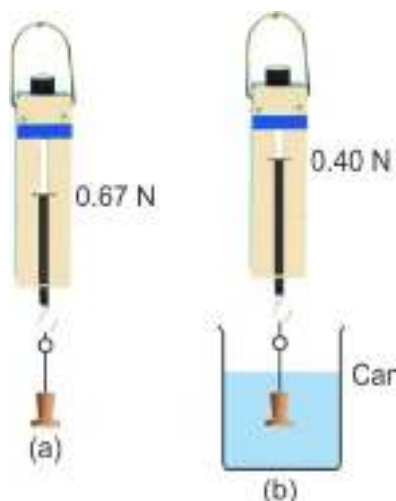
Solution

A piece of wood if left under water comes to the surface of water because the upthrust on body due to its submerged part is equal to its own weight.

8.Describe an experiment to show that a body immersed in a liquid appears lighter than it really is.

Solution

Experiment to show that a body immersed in a liquid appears lighter:



Take a solid body and suspend it by a thin thread from the hook of a spring balance as shown in the above figure (a). Note its weight. Above figure (a) shows the weight as 0.67 N.

Then, take a can filled with water. Immerse the solid gently into the water while hanging from the hook of the spring balance as shown in figure (b). Note its weight. Above figure (b) shows the weight as 0.40 N.

The reading in this case (b) shall be less than the reading in the case (a), which proves that a body immersed in a liquid appears to be lighter.

9. Will a body weigh more in air or vacuum when weighed with a spring balance? Give a reason for your answer.

Solution

A body shall weigh more in vacuum because in vacuum, i.e. in absence of air, no upthrust will act on the body.

10. A metal solid cylinder tied to a thread is hanging from the hook of a spring balance. The cylinder is gradually immersed into the water contained in a jar. What changes do you expect in the readings of the spring balance? Explain your answer.

Solution

The readings in the spring balance decreases.

As the cylinder is immersed in the jar of water, an upward force acts on it, which is in opposition to the weight component of the cylinder. Hence the cylinder appears to be lighter.

11. A body dipped into a liquid experiences an upthrust. State two factors on which upthrust on the body depends.

Solution

Upthrust on a body depends on the following factors:

- (i) Volume of the body submerged in the liquid or fluid.
- (ii) Density of liquid or fluid in which the body is submerged.

12. How is the upthrust related to the volume of the body submerged in a liquid?

Solution

Larger the volume of body submerged in liquid, greater is the upthrust acting on it.

13. A bunch of feathers and a stone of the same mass are released simultaneously in air. Which will fall faster and why? How will your observation be different if they are released simultaneously in vacuum?

Solution

When a bunch of feathers and a stone of the same mass are released simultaneously in air, the feathers will fall after the stone falls due to air friction. In vacuum, as there is no air friction, the acceleration due to gravity of both bodies will be the same, and therefore, the feathers and the stone will fall at the same time.

14. A body experiences an upthrust F_1 in river water and F_2 in sea water when dipped up to the same level. Which is more, F_1 or F_2 ? Give reason.

Solution

$F_2 > F_1$; Sea water is denser than river water; therefore, the upthrust due to sea water will be greater than that due to river water at the same level. This shall make the body to appear lighter in the sea water.

15. A small block of wood is completely immersed in (i) water, (ii) glycerine and then released. In each case, What do you observe? Explain the difference in your observation in the two cases.

Solution

Observation: Volume of a block of wood immersed in glycerine is smaller as compared to the volume of block immersed in water.

Explanation: Density of glycerine is more than that of water. Hence, glycerine exerts more upthrust on the block of wood than water, causing it to float in glycerine with a smaller volume.

16. A body of volume V and density ρ is kept completely immersed in a liquid of density ρ_L . If g is the acceleration due to gravity, then write expressions for the following:

- (i) The weight of the body, (ii) The upthrust on the body,
- (iii) The apparent weight of the body in liquid, (iv) The loss in weight of the body.

Solution

(i) Weight of the body = $V\rho g$

(ii) Upthrust on the body = $V\rho_L g$

(iii) Apparent weight of the body in liquid = $V(\rho - \rho_L)g$

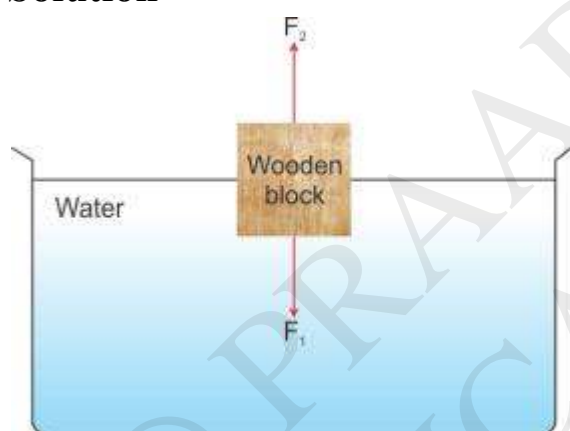
(iv) Loss in weight of the body = $V\rho_L g$

17. A body held completely immersed inside a liquid experiences two forces:

- (i) F_1 , the force due to gravity and
- (ii) F_2 , the buoyant force.

Draw a diagram showing the direction of these forces acting on the body and state the condition when the body will float or sink.

Solution



If $F_1 < F_2$ or $F_1 = F_2$, the body will float.

If $F_1 > F_2$, the body will sink.

18. Complete the following sentences:

(a) Two balls, one of iron and the other of aluminium experience the same upthrust when dipped completely in water if _____.

(b) An empty tin container with its mouth closed has an average density equal to that of a liquid. The container is taken 2 m below the surface of that liquid and is left there. Then the container will _____.

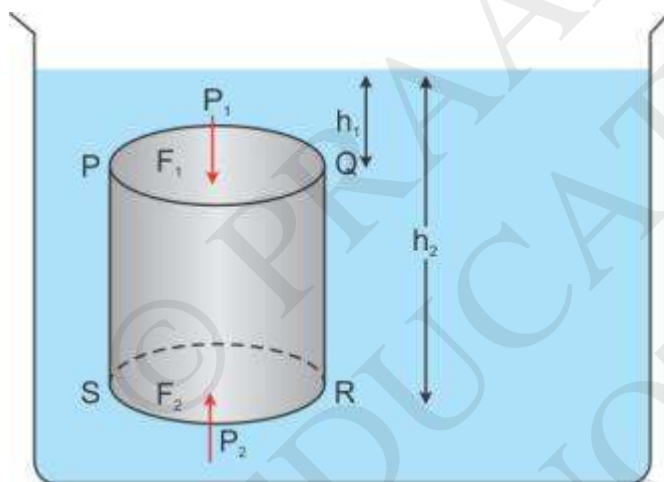
(c) A piece of wood is held under water. The upthrust on it will be _____ the weight of the wood piece.

Solution

- (a) Both have equal volumes. (b) Bounce back to the surface.
(c) More than

19. Prove that the loss in weight of a body when immersed wholly or partially in a liquid is equal to the buoyant force (or upthrust) and this loss is because of the difference in pressure exerted by liquid on the upper and lower surfaces of the submerged part of body.

Solution



Consider a cylindrical body PQRS of cross-sectional area A immersed in a liquid of density ρ as shown in the figure above. Let the upper surface PQ of the body is at a depth h_1 while its lower surface RS is at depth h_2 below the free surface of liquid.

At depth h_1 , the pressure on the upper surface PQ,

$$P_1 = h_1 \rho g.$$

Therefore, the downward thrust on the upper surface PQ,

$$F_1 = \text{Pressure} \times \text{Area} = h_1 \rho gA \dots\dots\dots(i)$$

At depth h_2 , pressure on the lower surface RS,

$$P_2 = h_2 \rho g$$

Therefore, the upward thrust on the lower surface RS,

$$F_2 = \text{Pressure} \times \text{Area} = h_2 \rho gA \dots\dots\dots(ii)$$

The horizontal thrust at various points on the vertical sides of body get balanced because the liquid pressure is the same at all points at the same depth.

From the above equations (i) and (ii), it is clear that $F_2 > F_1$ because $h_2 > h_1$ and therefore, body will experience a net upward force.

Resultant upward thrust or buoyant force on the body,

$$\begin{aligned} F_B &= F_2 - F_1 \\ &= h_2 \rho g A - h_1 \rho g A \\ &= A (h_2 - h_1) \rho g \end{aligned}$$

However, $A (h_2 - h_1) = V$, the volume of the body is submerged in a liquid.

Therefore, upthrust $F_B = V \rho g$.

Now, $V \rho g = \text{Volume of solid immersed} \times \text{Density of liquid} \times \text{Acceleration due to gravity}$

$= \text{Volume of liquid displaced} \times \text{Density of liquid} \times \text{Acceleration due to gravity}$

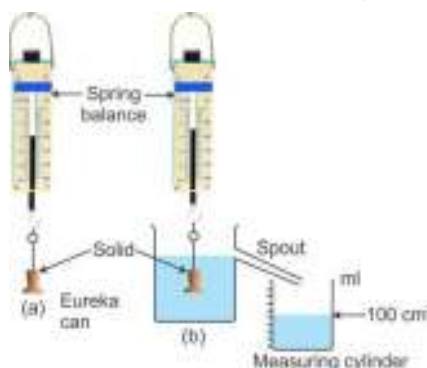
$= \text{Mass of liquid displaced} \times \text{Acceleration due to gravity}$

$= \text{Weight of the liquid displaced by the submerged part of the body}$

Thus, Upthrust $F_B = \text{weight of the liquid displaced by the submerged part of the body} \dots (iii)$

Now, let us take a solid and suspend it by a thin thread from the hook of a spring balance and note its weight.

Then take a eureka can and fill it with water up to its spout. Arrange a measuring cylinder below the spout of the eureka can as shown. Immerse the solid gently in water. The water displaced by the solid is collected in the measuring cylinder.



When the water stops dripping through the spout, note the weight of the solid and volume of water collected in the measuring cylinder.

From the diagram, it is clear that

Loss in weight (Weight in air - Weight in water) = Volume of water displaced.

Or, Loss in weight = Volume of water displaced $\times 1 \text{ gcm}^{-3}$ [Because the density of water = 1 gcm^{-3}]

Or, Loss in weight = Weight of water displaced(iv)

From equations (iii) and (iv),

Loss in weight = Upthrust or buoyant force

20. A sphere of iron and another sphere of wood of the same radius are held under water. Compare the upthrust on the two spheres.

Hint: Both have equal volume inside the water.

Solution

Since the spheres have the same radius, both will have an equal volume inside water, and hence, the upthrust acted by water on both the spheres will be the same.

Hence, the required ratio of upthrust acting on two spheres is 1:1.

21. A sphere of iron and another of wood, both of same radius are placed on the surface of water. State which of the two will sink? Give a reason for your answer.

Solution

Sphere of iron will sink.

Density of iron is more than the density of water, so the weight of iron sphere will be more than the upthrust due to water in it; thus, it causes the iron sphere to sink.

Density of wood is less than the density of water, so the weight of sphere of wood shall be less than the upthrust due to water in it. So, the sphere of wood will float with a volume submerged inside water which is balanced by the upthrust due to water.

22. How does the density of material of a body determine whether it will float or sink in water?

Solution

The bodies of average density greater than that of the liquid sink in it. While the bodies of average density equal to or smaller than that of liquid float on it.

23. A body of density ρ is immersed in a liquid of density ρ_L . State the condition when the body will (i) float and (ii) sink in the liquid.

Solution

- (i) The body will float if $\rho < \text{or } = \rho_L$
- (ii) The body will sink if $\rho > \rho_L$

24. It is easier to lift a heavy stone under water than in air. Explain.

Solution

It is easier to lift a heavy stone under water than in air because in water, it experiences an upward buoyant force which balances the actual weight of the stone acting downwards. Thus, due to upthrust there is an apparent loss in the weight of the heavy stone, which makes it lighter in water, and hence easy to lift.

25.State the Archimedes' principle.

Solution

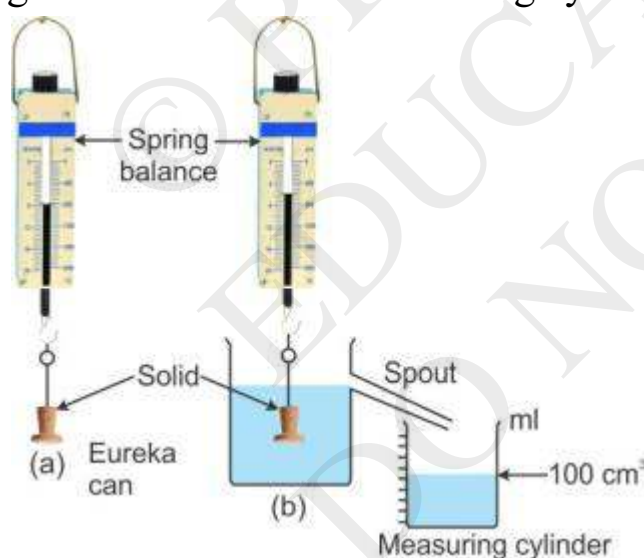
Archimedes' principle states that when a body is immersed partially or completely in a liquid, it experiences an upthrust, which is equal to the weight of liquid displaced by it.

26.Describe an experiment to verify the Archimedes' principle.

Solution

Let us take a solid and suspend it by a thin thread from the hook of a spring balance and note its weight (Fig a).

Then take a eureka can and fill it with water up to its spout. Arrange a measuring cylinder below the spout of the eureka can as shown. Immerse the solid gently in water. The water displaced by the solid gets collected in the measuring cylinder.



When water stops dripping through the spout, note the weight of the solid and volume of water collected in the measuring cylinder.

From diagram, it is clear that

Loss in weight (Weight in air- weight in water) = 300 gf - 200 gf = 100 gf

Volume of water displaced = Volume of solid = 100 cm³

Because density of water = 1 gcm⁻³

Weight of water displaced = 100 gf = Upthrust or loss in weight
This verifies Archimedes' principle.

MULTIPLE CHOICE TYPE :

1. A body will experience minimum upthrust when it is completely immersed in which of the following liquids:

- (a) Turpentine (b) Water
(c) Glycerine (d) Mercury

Solution

Turpentine

2. The S.I. unit of upthrust is given by the following unit:

- (a) Pa (b) N
(c) kg (d) kg m^2

Solution

N

3. A body of density ρ sinks in a liquid of density ρ_L . The densities ρ and ρ_L are related as shown below:

- (a) $\rho = \rho_L$ (b) $\rho < \rho_L$
(c) $\rho > \rho_L$ (d) Nothing can be said.

Solution

$\rho > \rho_L$

Numericals:

1. A body of volume 100 cm^3 weighs 5 kgf in air. It is completely immersed in a liquid of density $1.8 \times 10^3 \text{ kg m}^{-3}$. Find:

- (i) The upthrust due to liquid and
- (ii) The weight of the body in liquid.

Solution

Volume of body = $1000 \text{ cm}^3 = 100 \times 10^{-1} \text{ m}^3$

Weight in air = 5 kgf

Density of liquid = $1.8 \times 10^3 \text{ kgm}^{-3}$

$$\begin{aligned} \text{(i) upthrust due to liquid} &= \text{volume of the solid submerged} \times \text{density} \\ &\quad \text{of liquid} \times g \\ &= 100 \times 10^{-1} \times 1.8 \times 10^3 \times g \\ &= 0.18 \text{ kgf} \end{aligned}$$

$$\begin{aligned} \text{(ii) weight of body in liquid} &= \text{weight of body in air} - \text{upthrust} \\ &= 5 \text{ kgf} - 0.18 \text{ kgf} \\ &= 4.82 \text{ kgf} \end{aligned}$$

2. A body weighs 450 gf in air and 310 gf when completely immersed in water. Find the following factors:

- (i) The volume of the body,
 - (ii) The loss in weight of the body, and (iii) The upthrust on the body.
- State the assumption made in part (i).

Solution

Weight of body in air = 450 gf

Weight of body in water = 310 gf

$$\begin{aligned} \text{(i) volume of the body} &= \text{loss in weight} \times \text{density of water} \\ &= (450 - 310) \times 1 \text{ [Assumption density of water} = 1 \text{ gcm}^3\text{]} \\ &= 140 \text{ cm}^3 \end{aligned}$$

$$\begin{aligned}
 \text{(ii) loss in weight} &= \text{weight of body in air} - \text{weight of body in water} \\
 &= (450-310)\text{gf} \\
 &= 140 \text{ gf}
 \end{aligned}$$

$$\text{(iii) upthrust on body} = \text{loss in weight} = 140 \text{ gf}$$

3. You are provided with a hollow iron ball A of volume 15 cm^3 and mass 12 g and a solid iron ball B of mass 12 g . Both are placed on the surface of water contained in a large tub.

(a) Find upthrust on each ball.

(b) Which ball will sink? Give a reason for your answer. (Density of iron = 8.0 g cm^{-3})

Solution

Volume of hollow iron ball A, = 15 cm^3

Mass of hollow iron ball A = 12 g

Mass of solid iron ball B = 12 g

Density of iron = 8.0 g cm^{-3}

$$\text{Volume of solid iron ball B} = \frac{\text{mass}}{\text{density}} = \frac{12}{8} = 1.5 \text{ cm}^3$$

$$\begin{aligned}
 \text{(a) Upthrust on ball A} &= \text{Volume of iron ball A} \times \text{Density of water} \times g \\
 &= 15 \times 1 \times g = 15 \text{ gf}
 \end{aligned}$$

$$\begin{aligned}
 \text{Upthrust on ball B} &= \text{Volume of iron ball A} \times \text{density of water} \times g \\
 &= 1.5 \times 1 \times g = 1.5 \text{ gf}
 \end{aligned}$$

(b) ball B will sink because the upthrust on ball B (= 1.5 gf) is less than its weight 12 gf , while the upthrust on ball A (= 15 gf) if it is fully submerged, which is greater than its weight 12 gf , so it will float with its that much part submerged for which upthrust becomes equal to 12 gf (its weight).

4. A solid of density 5000 kg m^{-3} weighs 0.5 kgf in air. It is completely immersed in water of density 1000 kg m^{-3} . Calculate the apparent weight of the solid in water.

Solution

Density of solid = 5000 kgm^{-3}

Weight of solid = 0.5 kgf

Density of water = 1000 kg m^{-3}

Here, upthrust = volume of the solid \times density of water $\times g$

$$\begin{aligned} &= \frac{\frac{0.5}{g}}{5000} \times 1000 \times g \\ &= \frac{0.5}{5000 \times g} \times 1000 \times g = 0.1 \text{ kgf} \end{aligned}$$

Apparent weight = trust weight – upthrust
 $= 0.5 - 0.1 = 0.4 \text{ kgf}$

5. Two spheres A and B, each of volume 100 cm^3 are placed on water (density = 1.0 g cm^{-3}). The sphere A is made of wood of density 0.3 g cm^{-3} and the sphere B is made of iron of density 8.9 g cm^{-3} .

(a) Find:

- (i) The weight of each sphere, and
 - (ii) The upthrust on each sphere.
- (b) Which sphere will float? Give reason.

Solution

Volume of sphere A & B = 100 cm^3

Density of water = 1 gcm^{-3}

Density of sphere A = 0.3 gcm^{-3}

Density of sphere B = 8.9 gcm^{-3}

(a) (i) weight of sphere A = (density of sphere A \times volume) $\times g$
 $= 0.3 \times 100 \times g = 30 \text{ gf}$

Weight of sphere B = (density of sphere B \times volume) $\times g$

$$= 8.9 \times 100 \times g = 890 \text{ gf}$$

$$\begin{aligned} \text{(ii) upthrust on sphere A} &= \text{volume of sphere A} \times \text{density of water} \times g \\ &= 100 \times 1 \times g = 100 \text{ gf} \end{aligned}$$

$$\begin{aligned} \text{Upthrust on sphere B} &= \text{volume of sphere B} \times \text{density of water} \times g \\ &= 100 \times 1 \times g = 100 \text{ gf} \end{aligned}$$

Since the volume of both sphere is same inside water , the upthrust acting on them will also be same .

(b) the sphere A will float because the density of wood is less than the density of water .

6. The mass of a block made of certain material is 13.5 kg and its volume is $15 \times 10^{-3} \text{ m}^3$.

(a) Calculate upthrust on the block if it is held fully immersed in water.

(b) Will the block float or sink in water when released? Give a reason for your answer.

(c) What will be the upthrust on block while floating?

Take density of water = 1000 kg m^{-3} .

Solution

Mass of a block = 13.5 kg

Weight of block = 13.5 kgf

Volume = $15 \times 10^{-3} \text{ m}^3$

Density of water = 1000 kgm^{-3}

$$\begin{aligned} \text{(a) upthrust} &= \text{volume of block} \times \text{density of water} \times g \\ &= 15 \times 10^{-3} \times 1000 \times g \\ &= 15 \text{ kgf} \end{aligned}$$

(b) the block will float since the upthrust on it is more than its Weight (= 13.5 kgf) when fully immersed in water .

(c) while floating upthrust = 13.5 kgf (weight of the body)

7. A piece of brass weighs 175 gf in air and 150 gf when fully submerged in water. The density of water is 1.0 g cm^3 .

(i) What is the volume of the brass piece? (ii) Why does the brass piece weigh less in water?

solution

weight of piece of brass in air = 175 gf

weight of piece of brass when fully immersed in water = 150 gf

density of water = 1 g cm^{-3}

(i) volume of brass piece = loss in weight = $175 - 150 = 25 \text{ cm}^3$

(ii) the brass piece weight less in water due to upthrust .

8. A metal cube of edge 5 cm and density 9 g cm^{-3} is suspended by a thread so as to be completely immersed in a liquid of density 1.2 g cm^{-3} . Find the tension in thread. (Take $g = 10 \text{ m s}^{-2}$)

Solution

Given side of the cube = 5 cm

\therefore volume of the cube = $5 \times 5 \times 5 = 125 \text{ cm}^3$

Mass of the cube = volume \times density
 $= 125 \times 9 = 1125 \text{ g}$

\therefore weight of the cube = 1125 gf (downwards)

Upthrust on cube = weight of the liquid displaced

$= \text{volume of the cube} \times \text{density of liquid} \times g$
 $= 125 \times 1.2 \times g$
 $= 150 \text{ gf (upwards)}$

Tension in thread = net downward force

$= \text{weight of cube} - \text{upthrust on cube}$
 $= 1125 - 150 = 975 \text{ gf} = 9.75 \text{ N}$

9. A block of wood is floating on water with its dimensions 50 cm x 50 cm x 50 cm inside water. Calculate the buoyant force acting on the block. Take $g = 9.8 \text{ N kg}^{-1}$.

Solution

Volume of block of wood = $50 \text{ cm} \times 50 \text{ cm} \times 50 \text{ cm} = 125000 \text{ cm}^3 = 0.125 \text{ m}^3$

Given , $g = 9.8 \text{ m/s}^2$

$$\begin{aligned}\text{Buoyant force} &= V\rho g \\ &= 0.125 \times 1000 \times 9.8 \text{ N} \\ &= 1225 \text{ N}\end{aligned}$$

10. A body of mass 3.5 kg displaces 1000 cm^3 of water when fully immersed inside it. Calculate: (i) the volume of body, (ii) the upthrust on body and (iii) the apparent weight of body in water.

Solution

Mass of body = 3.5 kg

Weight of the body = 3.5 kgf

Volume of water displacement when body is fully immersed = 1000 cm^3

(i) volume of body when fully immersed in liquid = volume of water displaced

$$\therefore \text{volume of body} = 1000 \text{ cm}^3 \text{ or } 0.001 \text{ m}^3$$

$$\begin{aligned}\text{(ii) upthrust on body} &= \text{volume of body} \times \text{density of water} \times g \\ &= 0.001 \times 1000 \times g \\ &= 1 \text{ kgf}\end{aligned}$$

$$\begin{aligned}\text{(iii) Apparent weight} &= \text{true weight} - \text{upthrust} \\ &= (3.5 - 1) \\ &= 2.5 \text{ kgf}\end{aligned}$$

EXERCISE –(B)

1. Define the term density.

Solution

The density of a substance is its mass per unit volume.

2. State (i) C.G.S. and (ii) S.I. units of density.

Solution

(i) The C.G.S. unit of density is gcm^{-3} .

(ii) The S.I. unit of density is kgm^{-3} .

3. Express the relationship between the C.G.S. and S.I. units of density.

Solution

$$1 \text{ gcm}^{-3} = 1000 \text{ kgm}^{-3}$$

4. 'The density of iron is 7800 kg m^{-3} '. What do you understand by this statement?

Solution

It means the mass of 1 m^{-3} of iron is 7800 kg.

5. Write the density of water at 4°C in S.I. units.

Solution

Density of water at 4°C in S.I. units is 1000 kgm^{-3} .

6. How are the (i) Mass, (ii) Volume and (iii) Density of a metallic piece affected, if at all, with an increase in temperature?

Solution

(i) Mass of a metallic body remains unchanged with increase in temperature.

(ii) Volume of metallic body increases with an increase in temperature.

(iii) Density = $\frac{\text{Mass}}{\text{volume}}$ of a metallic body decreases with an increase in temperature.

7. Water is heated from 0°C to 10°C. How does the density of water change with temperature?

Solution

On heating from 0°C the density of water increases up to 4°C and then decreases beyond 4°C

8. Complete the following sentences.

(i) Mass = × density

(ii) S.I. unit of density is

(iii) Density of water is kg m⁻³.

(iv) Density in kg m⁻³ = × density in g cm⁻³

Solution

(i) Volume, (ii) kg m⁻³, (iii) 1000 and (iv) 1000

9. What do you understand by the term relative density of a substance?

Solution

The relative density of a substance is the ratio of density of that substance to the density of water at 4°C .

10. What is the unit of relative density?

Solution

Relative density is the ratio of two similar quantities; thus, it has no unit.

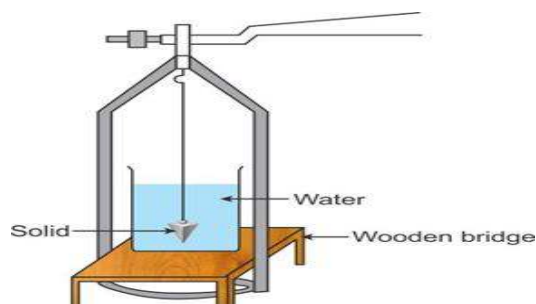
11. Differentiate between density and relative density of a substance.

Solution

Density of a substance is the ratio of its mass to its volume but R.D. of a substance is the ratio of density of that substance to the density of water at 4°C

12. With the use of Archimedes' principle, state how you will find relative density of a solid denser than water. How will you modify your experiments if the solid is soluble in water?

Solution



Steps:

(i) With the help of a physical balance, find the weight, W_1 of the given solid.

(ii) Immerse the solid completely in a beaker filled with water such that it does not touch the walls and bottom of beaker, and find the weight W_2 of solid in water.

Observations:

Loss in weight of solid when immersed in water = $(W_1 - W_2)$ gf

$$\text{R.D.} = \frac{\text{Weight of solid in air}}{\text{Loss of weight of solid in water}}$$

$$\text{R.D.} = \frac{W_1}{W_1 - W_2}$$

If the solid is soluble in water, then instead of water, take a liquid in which the solid is insoluble and it sinks in the liquid.

$$\text{Then, R.D.} = \frac{\text{Weight of solid in air}}{\text{Loss of weight of solid in liquid}} \times \text{R.D. of the liquid}$$

13. A body weighs W gf in air and W_1 gf when it is completely immersed in water. Find: (i) Volume of the body, (ii) Upthrust on the body and (iii) Relative density of the material of the body.

Solution

(i) volume of the body = $(W - W_1) \text{ cm}^3$

(ii) upthrust on the body = $(W - W_1) \text{ gf}$

(iii) R.D. of the material of body = $\frac{W}{W - W_1}$

14. Describe an experiment, using Archimedes principle, to find relative density of a liquid.

Solution

Relative density is the ratio of weight of a given volume of liquid to the weight of the same volume of water.

Using Archimedes principle, we can perform an experiment which measures the weight of a liquid displaced by a body and weight of water displaced by the same body.

Weight of liquid displaced by a body is given by the difference of weight of a body in air and weight of a body in liquid.

Weight of the water displaced by the body can be found by knowing the difference of the weight of the body in air and the weight of the body in water.

Therefore, using Archimedes principle, the relative density can be calculated using the formula:

$$\text{RD of liquid} = \frac{w_1 - w_2}{w_1 - w_3}$$

15. A body weighs $W_1\text{gf}$ in air and when immersed in a liquid it weighs $W_2\text{gf}$, while it weighs $W_3\text{gf}$ on immersing it in water. Find: (i) volume of the body (ii) upthrust due to liquid (iii) relative density of the solid and (iv) relative density of the liquid.

Solution

(i) Volume of the body = $W_1 - W_3 \text{ cm}^3$

(ii) Upthrust due to liquid = loss in weight when immersed in liquid
= $W_1 - W_2 \text{ gf}$

(iii)

Weight of a body in air = $W_1\text{gf}$

Weight of that body in liquid = $W_2\text{gf}$

Weight of that body in water = $W_3\text{gf}$

$$\begin{aligned}\text{RD of solid} &= \frac{\text{weight of solid in air}}{\text{weight in air} - \text{weight in water}} \\ &= \frac{w_1}{w_1 - w_2}\end{aligned}$$

(iv)

Weight of a body in air = $W_1 \text{gf}$

Weight of that body in liquid = $W_2 \text{gf}$

Weight of that body in water = $W_3 \text{gf}$

$$\text{RD of liquid} = \frac{w_1 - w_2}{w_1 - w_3}$$

MULTIPLE CHOICE TYPE :

1. Relative density of a substance is expressed by comparing the density of that substance with the density of the substance listed below:

(a) Air (b) Mercury (c) Water (d) Iron

Solution

Water

2. The unit of relative density is which of the following listed below:

(a) g cm^{-3} (b) kg m^{-3} (c) $\text{m}^3 \text{kg}^{-1}$ (d) no unit

Solution

No unit.

3. The density of water is as listed below:

(a) 1000 g cm^{-3} (b) 1 kg m^{-3} (c) 1 g cm^{-3} (d) None of these.

Solution

1 g cm^{-3}

NUMERICAL :

1. The density of copper is 8.83 g cm^{-3} . Express it in kg m^{-3} .

Solution

Density of copper in C.G.S = 8.83 gcm^{-3}

Density of copper in S.I = $\frac{8.83}{1000 \times 100^{-6}} = 8830 \text{ kgm}^{-3}$

2. The relative density of mercury is 13.6. State its density in (i) C.G.S. unit and (ii) S.I. unit.

Solution

R.D. of mercury = 13.6

(i) density in C.G.S = 13.6 gcm^{-3}

(ii) density in S.I = $13.6 \times 10^3 \text{ kgm}^{-3}$

3. The density of iron is $7.8 \times 10^3 \text{ kg m}^{-3}$. What is its relative density?

Solution

Density of iron = $7.8 \times 10^3 \text{ kgm}^{-3}$

Density of iron in C.G.S. = 7.8 gcm^{-3}

R.D. = density in C.G.S. (without unit) = 7.8

4. The relative density of silver is 10.8. Find its density.

Solution

R.D. of silver = 10.8

Density of silver in C.G.S = 10.8 gcm^{-3}

Density in S.I. = $10.8 \times 10^3 \text{ kgm}^3$

5. Calculate the mass of a body whose volume is 2 m^3 and relative density is 0.52.

Solution

R.D. of silver = 0.52

Volume = 2 m^3

Density of body in S.I = $0.52 \times 10^3 \text{ kgm}^3$

$\therefore \text{mass} = \text{density} \times \text{volume} = (0.52 \times 10^3) \times 2 = 1040 \text{ kg}$

6. Calculate the mass of air in a room of dimensions $4.5 \text{ m} \times 3.5 \text{ m} \times 2.5 \text{ m}$ if the density of air at N.T.P. is 1.3 kgm^{-3} .

Solution

Volume of air = $4.5 \times 3.5 \times 2.5 \text{ m}^3$

Density of air at NTP = 1.3 kgm^3

Mass of air = density \times volume

Or mass = $(1.3) \times (4.5 \times 3.5 \times 2.5) = 51.19 \text{ kg}$

7. A piece of stone of mass 113 g sinks to the bottom in water contained in a measuring cylinder and water level in cylinder rises from 30 ml to 40 ml. Calculate R.D. of stone.

Solution

Mass of stone = 113g

Rise in water level = $(40 - 30) \text{ ml} = 10 \text{ ml}$

This rise is equal to the space occupied (volume) by the stone

$\therefore \text{volume of stone} = 10 \text{ cm}^3$

Density of stone in C.G.S = $\frac{\text{mass}}{\text{volume}} = \frac{113}{10} = 11.3 \text{ gcm}^{-3}$

R.D. = 11.3

8. A body of volume 100 cm^3 weighs 1 kgf in air. Find: (i) Its weight in water and (ii) Its relative density.

Solution

volume of body = 100 cm^3

weight in air $w_1 = 1 \text{ kgf} = 1000 \text{ gf}$

mass of body = $1 \text{ kg} = 1000 \text{ g}$

R.D. of solid = 10

R.D. of water = 1

(i) let w_2 be the weight of the body in water

$$\text{R.D. of body} = \frac{w_1}{w_1 - w_2} \times \text{R.D. of water}$$

$$\text{Or } 10 = \frac{1000}{(1000 - w_2)} \times 1$$

$$\text{Or } 10 (1000 - w_2) = 1000$$

$$\text{Or } 1000 - w_2 = 100$$

$$\text{Or } w_2 = 900 \text{ gf}$$

(ii) R.D. of body = density in C.G.S (without unit)

$$\text{Or R.D.} = \frac{\text{mass}}{\text{volume}} = \frac{1000}{100} = 10$$

9. A body of mass 70 kg , when completely immersed in water, displaces $20,000 \text{ cm}^3$ of water. Find: (i) The mass of body in water and (ii) The relative density of material of the body.

Solution

Mass of body = 70 kg

Volume of water displaced by body = $20000 \text{ cm}^3 = 0.02 \text{ m}^3$

(i) mass of solid immersed in water = mass of water displaced

Mass of solid immersed in water = density of water \times volume of water displaced

$$\text{Mass of solid immersed in water} = 1000 \text{ kgm}^{-3} \times 0.02 \text{ m}^3 = 20 \text{ kg}$$

(ii) R.D. of solid = density in C.G.S (without unit)

$$\text{Density in C.G.S.} = \frac{\text{mass}}{\text{volume}} = \frac{70 \times 1000}{20000} = 3.5 \text{ gcm}^3$$

$$\text{R.D.} = 3.5$$

10. A solid weighs 120 gf in air and 105 gf when it is completely immersed in water. Calculate the relative density of solid.

Solution

Weight of solid in air $W_1 = 120 \text{ gf}$

Weight of solid when completely immersed in water $W_2 = 105 \text{ gf}$

$$\text{R.D. of solid} = \frac{W_1}{W_1 - W_2} \times \text{R.D. of water}$$

$$\text{R.D. of solid} = \frac{120}{120 - 105} \times 1$$

$$\text{R.D. of solid} = 8$$

11. A solid weighs 32 gf in air and 28.8 gf in water. Find: (i) The volume of solid, (ii) R.D. of solid and (iii) The weight of solid in a liquid of density 0.9 g cm^{-3} .

Solution

Weight of solid in air $W_1 = 32 \text{ gf}$

Weight of solid when completely immersed in water $W_2 = 28.8 \text{ gf}$

$$\text{(i) volume of solid} = \frac{\text{mass}}{\text{density of solid}} = \frac{32}{10} = 3.2 \text{ m}^3$$

$$\text{(ii) R.D. of solid} = \frac{W_1}{W_1 - W_2} \times \text{R.D. of water}$$

$$\text{R.D. of solid} = \frac{32}{32 - 28.8} \times 1$$

$$\text{R.D. of solid} = 10$$

$$\text{(iii) weight of solid in liquid of density } 0.9 \text{ gcm}^{-3} = W_3$$

$$\text{R.D. of solid} = \frac{W_1}{W_1 - W_3} \times \text{R.D. of liquid}$$

$$\text{Or, } 10 = \frac{32}{32 - W_3} \times 0.9$$

$$\text{Or } W_3 = 29.12 \text{ gf}$$

12. A body weighs 20 gf in air and 18.0 gf in water. Calculate the relative density of the material of the body.

Solution

Weight of body in air $W_1 = 20\text{gf}$

Weight of body when completely immersed in water $W_2 = 18 \text{ gf}$

$$\text{R.D. of body} = \frac{W_1}{W_1 - W_3} \times \text{R.D. of water}$$

$$\text{R.D. of body} = \frac{20}{20 - 18} \times 1$$

$$\text{R.D. of body} = 10$$

13. A solid weighs 1.5 kgf in air and 0.9 kgf in a liquid of density $1.2 \times 10^3 \text{ kg m}^{-3}$. Calculate R.D. of solid.

Solution

Weight of body in air $W_1 = 1.5\text{kgf}$

Weight of body when completely immersed in liquid $w_2 = 0.9 \text{ kgf}$

Density of liquid $= 1.2 \times 10^3 \text{ kgm}^{-3}$

R.D. of liquid $= 1.2$

$$\text{R.D. of solid} = \frac{W_1}{W_1 - W_3} \times \text{R.D. of liquid}$$

$$\text{R.D. of solid} = \frac{1.5}{1.5 - 0.9} \times 1.2$$

$$\text{R.D. of solid} = 3$$

14. A jeweller claims that he makes ornaments of pure gold that has a relative density of 19.3. He sells a bangle weighing 25.25 gf to a person. The clever customer weighs the bangle when immersed in water and finds that it weighs 23.075 gf in water. With the help of suitable calculations, find out whether the ornament is made of pure gold or not.

Hint : Calculate R.D. of the material of the bangle.

Solution

R.D. of pure gold = 19.3

Weight of bangle in air $W_1 = 25.25$ gf

Weight of bangle when completely immersed in water $W_2 = 23.075$ gf

$$\text{R.D. of bangle} = \frac{W_1}{W_1 - W_2} \times \text{R.D. of water}$$

$$\text{R.D. of bangle} = \frac{25.25}{25.25 - 23.075} \times 1$$

$$\text{R.D. of bangle} = 11.6$$

The bangle is not made of pure gold as its density is not 19.3

15. A piece of iron weighs 44.5 gf in air. If the density of iron is 8.9×10^3 , find the weight of iron piece when immersed in water.

Solution

Density of iron = $8.9 \times 10^3 = 8900$

Density of water = 1000

Weight of iron when immersed in water is given by

Weight of iron in water = weight of iron in air \times

$$\left[1 - \frac{\text{density of water}}{\text{density of iron}} \right]$$

$$= 44.5 \text{ gf} \times \left[1 - \frac{1000}{8900} \right]$$

$$= 39.5 \text{ kgf}$$

16. A piece of stone of mass 15.1 g is first immersed in a liquid and it weighs 10.9 gf. Then on immersing the piece of stone in water, it weighs 9.7 gf. Calculate:

- a. The weight of the piece of stone in air,
- b. The volume of the piece of stone,
- c. The relative density of stone,
- d. The relative density of the liquid.

Solution

a. the mass of stone is 15.1 g . hence , its weight in air will be $W_a = 15.1 \text{ gf}$

b. when stone is immersed in water its weight becomes 9.7 gf . so , the upthrust on the stone is $15.1 - 9.7 = 5.4 \text{ gf}$, since the density of water is 1 g cm^{-3} , the volume of stone is 5.4 cm^3

c. weight of stone in liquid is $W_l = 10.9 \text{ gf}$

weight of stone in water is $W_w = 9.7 \text{ gf}$

therefore the relative density of stone is

$$R.D_{\text{stone}} = \frac{W_A}{W_A - W_W} = \frac{15.1 \text{ gf}}{15.1 - 9.7 \text{ gf}}$$

$$\therefore R.D_{\text{stone}} = \frac{15.1}{5.4} = 2.8$$

d. Relative density of liquid is

$$R.D_{\text{liquid}} = \frac{W_A - W_L}{W_A - W_W} = \frac{15.1 - 10.9}{15.1 - 9.7} = \frac{4.2}{5.4}$$

$$\therefore R.D_{\text{stone}} = 0.7777 \approx 0.78$$

EXERCISE – (C)

1. State the principle of floatation.

Solution

According to the principle of floatation, the weight of a floating body is equal to the weight of the liquid displaced by its submerged part.

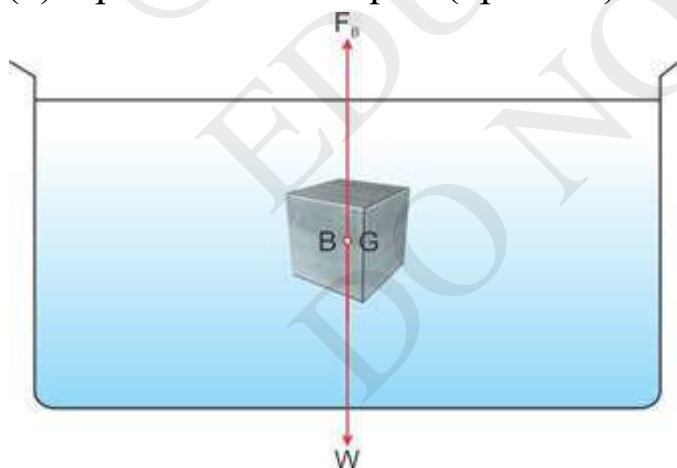
2. A body is held immersed in a liquid. (i) Name the two forces acting on the body and draw a diagram to show these forces. (ii) State how the magnitudes of two forces mentioned in part (i) determine whether the body will float or sink in liquid when it is released. (iii) What is the net force on the body if it (a) sinks and (b) floats?

Solution

(i) Two forces acting on the body are as listed below:

(a) Weight of the body (downwards)

(b) Upthrust of the liquid (upwards)



(ii) If the weight of the body is greater than the upthrust acting on it, the body will sink

If the weight of the body is equal to or less than the upthrust acting on it, the body will float.

- (iii) (a) The net force acting on the body when it sinks is body's own weight.
(b) The net force acting on the body when it floats is the upthrust due to the liquid.

3. When a piece of wood is suspended from the hook of a spring balance, it reads 70 gf. The wood is now lowered into water. What reading do you expect on the scale of spring balance?

Solution

The reading on the spring balance will be zero because wood floats on water and while floating the apparent weight = 0.

4 .A solid iron ball of mass 500 g is dropped in mercury contained in a beaker. (a) Will the ball float or sink? Give reasons. (b) What will be the apparent weight of the ball?

Solution

(a) The ball will float because the density of ball (i.e. iron) is less than the density of mercury.

(b) While floating, the apparent weight = 0.

5. How does the density of a substance determine whether a solid piece of that substance will float or sink in a given liquid?

Solution

The body will float if its density is less than or equal to the density of the liquid. $\rho_s \leq \rho_l$

The body will sink if its density is greater than the density of the liquid. $\rho_s > \rho_l$

6.Explain why an iron nail floats on mercury, but it sinks in water.

Hint: Density of iron is less than that of mercury, but more than that of water.

Solution

Density of iron is less than the density of mercury; hence, an iron nail floats in mercury and density of iron is more than the density of water; hence, an iron nail sinks in water.

7.A body floats in a liquid with a part of it submerged inside the liquid. (i) Is the weight of floating body greater than, equal to or less than the upthrust? (ii) What is the apparent weight of the floating body?

Solution

(i) Weight of the floating body is equal to the upthrust.

(ii) While floating, the apparent weight is zero.

8.A homogeneous block floats on water (a) partly immersed (b) completely immersed. In each case state the position of centre of buoyancy B with respect to the centre of gravity G of the block.

Solution

When the body is partially immersed, its centre of buoyancy will be below the centre of gravity of the block.

When the body is completely immersed, its centre of buoyancy will coincide the centre of gravity.

9.Fig. shows the same block of wood floating in three different liquids A, B and C of densities ρ_1 , ρ_2 and ρ_3 respectively. Which of the liquid has the highest density? Give a reason for your answer.



Solution

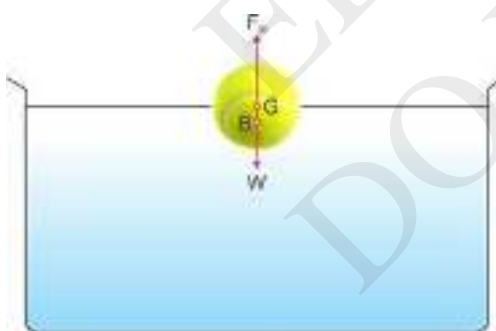
The upthrust on the body by each liquid is the same and equal to the weight of the body.

However, upthrust = Volume submerged $\times \rho \times g$,

For the liquid C, since the volume submerged is least so the density ρ_3 must be maximum.

10.Draw a diagram to show the forces acting on a body floating in water with its some part submerged. Name the forces and show their points of application. How is the weight of water displaced by the floating body related to the weight of the body itself?

Solution



The forces acting are as listed below:

- (i) Weight of the body acting downwards.
- (ii) Upthrust due to water acting upwards.

Weight of water displaced by the floating body = Weight of the body

11.What is the centre of buoyancy? State its position for a floating body with respect to the centre of gravity of the body.

Solution

Centre of buoyancy: It is the point through which the resultant of the buoyancy forces on a submerged body act; it coincides with the centre of gravity of the displaced liquid, if the body is completely immersed. For a floating body with its part submerged in the liquid, the centre of buoyancy is at the centre of gravity of the submerged part of the body and it lies vertically below the centre of gravity of the entire body.

12.A balloon filled with helium gas floats in a big closed jar which is connected to an evacuating pump. What will be your observation, if air from the jar is pumped out? Explain your answer.

Solution

Observation : The balloon will sink.

Explanation : As air is pumped out from jar, the density of air in jar decreases, so the upthrust on balloon decreases. As weight of balloon exceeds the upthrust on it, it sinks.

13.A block of wood is so loaded that it just floats in water at room temperature. What change will occur in the state of floatation, if (a) Some salt is added to water, (b) Water is heated? Give reasons.

Solution

(a) It will float with some part outside water.

Reason : On adding some salt to water, the density of water increases, so upthrust on a block of wood increases, and hence, the block rises

up till the weight of salty water displaced by the submerged part of block becomes equal to the weight of the block.

(b) The block will sink.

Reason: On heating, the density of water decreases, so upthrust on the block decreases and the weight of block exceeds upthrust due to which it sinks.

14. A body of volume V and density, ρ_s floats with volume v inside a liquid of density. ρ_l Show that $\frac{v}{V} = \frac{\rho_s}{\rho_l}$

Solution

Let V be the volume of a body of density ρ_s

Let the body be floating with its volume V immersed inside a liquid of density ρ_l

Then, weight of the body,

$W = \text{volume of body} \times \text{density of body} \times g$

Or, $W = V \rho_s g$

Weight of liquid displaced by body or upthrust

$F_B = \text{volume of displaced liquid} \times \text{density of liquid} \times g$

Or $F_B = V \rho_L g$

For floatation $W = F_B$

i.e, $V \rho_s g = V \rho_L g$

or $\frac{v}{V} = \frac{\rho_s}{\rho_L}$

thus $\frac{\text{volume of immersed part of body}}{\text{total volume of body}} = \frac{\text{density of body}}{\text{density of liquid}}$

15. Two identical pieces, one of ice (density = 900kg per meter cube) and other wood (density = 300kg per meter cube) float on water.

- a. Which of the two will have more volume submerged inside water**
- b. Which of two will experience more upthrust due to water.**

Solution

- a. Ice will be more submerged inside water. Ice has a greater density than wood, although the volume of both is the same. So to support a greater amount of weight, ice needs to displace more water, and to displace more water, it has to be submerged more as compared to wood.
- b. As ice displaces more water, it will experience more upthrust.

16. Why is the floating ice less submerged in brine than in water?

Solution

Density of brine is more than the density of water. Hence, the upthrust exerted by brine is more than the upthrust exerted by water on ice. Therefore, floating ice is less submerged in brine.

17. A man first swims in sea water and then in river water. (i) Compare the weights of sea water and river water displaced by him.

(ii) Where does he find it easier to swim and why?

Solution

(i) 1:1; The weight of the water displaced by the man in sea and river will be same and will be equal to his own weight.

(ii) He finds it easier to swim in the sea because the density of sea water is more than the density of river water. So his weight is balanced in sea water with a part of his body submerged in the water.

18. An iron nail sinks in the water while an iron ship floats on water. Explain the reason.

Solution

An iron nail sinks in water because density of iron is more than the density of water, so the weight of the nail is more than the upthrust of water on it.

On the other hand, ships are also made of iron, but they do not sink. This is because the ship is hollow and the empty space in it contains air, which makes its average density less than that of water. Therefore, even with a small portion of ship submerged in water, the weight of water displaced by the submerged part of ship becomes equal to the total weight of ship and it floats.

19. What can you say about the average density of a ship floating on water in relation to the density of water?

Solution

Due to the hollow and empty space in the ship, the average density of a ship is less than the density of water.

20 A piece of ice floating in a glass of water melts, but the level of water in the glass does not change.

Give reasons.

Hint: Ice contracts on melting.

Solution

When a floating piece of ice melts into water, it contracts by the volume equal to the volume of ice pieces above the water surface while floating on it. Hence, the level of water does not change when ice floating on it melts.

21.A body is held inside water contained in a vessel by tying it with a thread to the base of the vessel. Name the three forces that keep the body in equilibrium, and state the direction in which each force acts.

Solution

Forces acting on the body are listed below:

- (i) Weight of the body vertically downwards.
- (ii) Upthrust of water on body vertically upwards.
- (iii) Tension in thread vertically downwards.

22.A loaded cargo ship sails from sea water to river water. List your observations.

Solution

A ship submerges more as it sails from sea water to river water.

Density of river water is less than the density of sea water. Hence, according to the law of floatation, to balance the weight of the ship, a greater volume of water is required to be displaced in river water of lower density.

23.Explain the following observations listed below:

- (a) Icebergs floating in sea are dangerous for ships.
- (b) An egg sinks in fresh water, but floats in a strong salt solution.

(c) Toy balloons filled with hydrogen rise to the ceiling, but if they are filled with carbon dioxide, then they sink to the floor.

(d) As a ship in harbour is being unloaded, it slowly rises higher in water.

(e) A balloon filled with hydrogen rises to a certain height and then stops rising further.

(f) A ship submerges more as it sails from sea water to river water.

Solution

(a) Icebergs are dangerous for ships as they may collide with them. Icebergs being lighter than water, float on water with a major part of their surfaces laying under the water surface and only a small part lies outside water. Thus, it becomes difficult for the driver of the ship to estimate the size of the iceberg.

(b) Density of a strong salt solution is more than the density of fresh water. Hence, the salt solution exerts a greater upthrust on the egg which balances the weight of the egg, so the egg floats in a strong salt solution but sinks in fresh water.

(c) Density of hydrogen is much less than the density of carbon dioxide. When a balloon is filled with hydrogen, the weight of the air displaced by an inflated balloon (i.e. upthrust) becomes more than the weight of a gas filled balloon, and hence, it rises. In case of a balloon filled with carbon dioxide, weight of the balloon becomes more than the upthrust of the air, and hence, it sinks to the floor.

(d) As a ship in harbor is unloaded, its weight decreases. As a result, it displaces less water, and the ship's hull rises in water till the weight of the water displaced balances the weight of the unloaded ship.

(e) The reason is that the density of air decreases with altitude. Therefore, as the balloon gradually goes up, the weight of the displaced air (i.e. upthrust) decreases. It keeps on rising as long as the upthrust exceeds its weight. When upthrust becomes equal to its weight, it stops rising.

(f) Density of river water is less than the density of sea water. Hence, according to the law of floatation, to balance the weight of the ship, a great volume of water is required to be displaced in river water having a comparatively lower density.

MULTIPLE CHOICE TYPE

1. For a floating body, its weight W and upthrust F_B on it are related as listed below:

- (a) $W > F_B$ (b) $W < F_B$
(c) $W = F_B$ (d) Nothing can be said.

Solution

$$W = F_B$$

2. A body of weight W is floating in a liquid. Its apparent weight will be observed as listed below:

- (a) Equal to W (b) Less than W
(c) Greater than W (d) Zero

Solution

Zero

3. A body floats in a liquid A of density ρ_1 with a part of it submerged inside the liquid, while in liquid B of density ρ_2 it is totally submerged inside the liquid. The densities ρ_1 and ρ_2 are related as shown below:

- (a) $\rho_1 = \rho_2$ (b) $\rho_1 < \rho_2$
(c) $\rho_1 > \rho_2$ (d) Nothing can be said

Solution

$$\rho_1 > \rho_2$$

NUMERICAL

1. A rubber ball floats on water with its $\frac{1}{3}$ rd volume outside water. What is the density of rubber?

Solution

Let the volume of the ball be V

$$\text{Volume of ball above the surface of water} = \frac{1}{3} V$$

$$\therefore \text{volume of ball immersed in water} = V - \frac{1}{3} V = \frac{2}{3} V$$

By the principle of floatation,

$$\frac{\text{volume of rubber ball immerse}}{\text{total volume of rubber ball}} = \frac{\text{density of rubber}}{\text{density of water}}$$

$$\text{Or, } \frac{2}{3} = \frac{\text{density of rubber}}{1000}$$

$$\text{Or, density of rubber ball} = 1000 \times \frac{2}{3} = 666.7 \text{ kgm}^3 \approx 667 \text{ kgm}^3.$$

2. A block of wood of mass 24 kg floats in water. The volume of wood is 0.032 m^3 . Find the following factors listed below:

(a) The volume of block below the surface of water, (b) The density of wood.

(Density of water = 1000 kg m^{-3})

Solution

Mass of block of wood = 24 kg

Volume of wood = 0.032 m^3

(a) upthrust = volume of block below the surface of water(V) \times density of liquid $\times g$

Now for floatation upthrust = weight of the body = 24 kgf

Or $24 \text{ kgf} = V \times 1000 \times g$

Or $V = \frac{24}{1000} = 0.024 \text{ m}^3$

(b) According to the law of floatation

$$\frac{\text{volume of the submerged block}}{\text{total volume of block}} = \frac{\text{density of wood}}{\text{density of water}}$$

Or, $\frac{0.024}{0.032} = \frac{\text{density of wood}}{1000}$

Or density of wood = $1000 \times \frac{0.024}{0.032} = 750 \text{ kgm}^3$

3. A wooden cube of side 10 cm has mass 700 g. What part of it remains above the water surface while floating vertically on water surface?

Solution

Mass of wooden cube = 700 g

Side of wooden cube = 10 cm

Volume of the wooden cube = 10^3 cm^3

Density of wooden cube = $\frac{\text{mass}}{\text{volume}} = 700 \times 10^{-3} \text{ gcm}^3$

(b) According to the law of floatation

$$\frac{\text{volume of the submerged cube (v)}}{\text{total volume of cube (v)}} = \frac{\text{density of wood}}{\text{density of water}}$$

Or $\frac{v}{10^3} = \frac{700 \times 10^{-3}}{1}$

Or volume of the submerged cube = $1000 \times 700 \times 10^{-3} = 700 \text{ cm}^3$

Or volume of the wooden cube above the water surface = $V - v$
 $= 1000 - 700 = 300 \text{ cm}^3$ or 3m

4. A piece of wax floats in brine. What fraction of its volume will be immersed?

Density of wax = 0.95 g cm^{-3} , Density of brine = 1.1 g cm^{-3} .

Solution

Density of wax (ρ_v) = 0.95 g cm^{-3}

Density of brine (ρ_n) = 1.1 g cm^{-3}

Let the total volume of piece of wax be V and the volume of immersed portion be v .

According to the law of floatation,

$$\frac{v}{V} = \frac{\rho_v}{\rho_n}$$

$$\text{Or } \frac{v}{V} = \frac{0.95}{1.1} = 0.86$$

$$\text{Or } v = 0.86 V$$

Thus wax floats with 0.86^{th} part of its volume above the surface brine

5. If the density of ice is 0.9 g cm^{-3} , then what portion of an iceberg will remain below the surface of water in sea? (Density of sea water = 1.1 g cm^{-3})

Solution

Density of ice (ρ_1) = 0.9 g cm^{-3}

Density of sea water (ρ_2) = 1.1 g cm^{-3}

Let the total volume of the iceberg be V and the volume of immersed portion be v .

According to the law of floatation ,

$$\frac{v}{V} = \frac{0.9}{1.1} = \frac{9}{11}$$

$$\text{Or } v = \frac{9}{11} V$$

Thus ice floating with $\frac{9}{11}$ th part of its volume above the surface sea water

6. A piece of wood of uniform cross section and height 15 cm floats vertically with its height 10 cm in water and 12 cm in spirit. Find the densities of wood and spirit.

Solution

Height of wooden piece = 15 cm

Height of wooden piece submerged in water = 10 cm

Height of wooden piece submerged in spirit = 12 cm

Note : since the wooden block is of uniform cross section height will be proportional to volume

Say density of wood = $p_{\text{wood}} \text{ gcm}^{-3}$

Say density of spirit = $p_{\text{spirit}} \text{ gcm}^{-3}$

According to the law of floatation

$$\frac{v}{V} = \frac{P_{\text{WOOD}}}{P_{\text{WATER}}} = \frac{P_{\text{WOOD}}}{1}$$

$$\text{Or } P_{\text{WOOD}} = 0.667 \text{ gcm}^{-3}$$

Again according to the law of floatation

$$\frac{v}{V} = \frac{P_{\text{WOOD}}}{P_{\text{spirit}}}$$

$$\text{Or } \frac{12}{15} = \frac{p_{\text{wood}}}{p_{\text{spirit}}}$$

$$\text{Or } \frac{12}{15} = \frac{0.667}{p_{\text{spirit}}}$$

$$\text{Or } p_{\text{spirit}} = \frac{12}{10} \times 0.667 = 0.80 \text{ gcm}^{-3}$$

7. A wooden block floats in water with two-third of its volume submerged. (a) Calculate the density of wood. (b) When the same block is placed in oil, three-quarters of its volume is immersed in oil. Calculate the density of oil.

Solution

Volume of wooden block submerged in water(v) = $\frac{2}{3} \times$ total volume
(V)

Volume of wooden block submerged in oil (v') = $\frac{3}{4} \times$ total volume
(V)

Say density of wood = $p_{\text{wood}} \text{ gcm}^{-3}$

Say density of oil = $p_{\text{oil}} \text{ gcm}^{-3}$

According to the law of floatation

$$\frac{v}{V} = \frac{P_{\text{WOOD}}}{P_{\text{WATER}}}$$

$$\frac{2}{3} = \frac{P_{\text{WOOD}}}{P_{\text{WATER}}} = \frac{P_{\text{WOOD}}}{1000}$$

$$\text{Or } p_{\text{wood}} = 1000 \times \frac{2}{3} = 667 \text{ kgm}^{-3}$$

Again according to the law of floatation

$$\frac{v'}{V} = \frac{p_{\text{wood}}}{p_{\text{oil}}}$$

$$\text{Or } \frac{3}{4} = \frac{667}{p_{\text{oil}}}$$

$$\text{Or } p_{\text{oil}} = \frac{4}{3} \times 667 = 889 \text{ kgm}^{-3}$$

8 .The density of ice is 0.92 g cm^{-3} and that of sea water is 1.025 g cm^{-3} . Find the total volume of an iceberg which floats with its volume 800 cm^3 above water.

Solution

Let V be the volume of the iceberg

Volume of iceberg above water = 800 cm^3

Volume of iceberg submerged in water = v

Density of ice (p_{ice}) = 0.92 gcm^{-3}

Density of sea water ($P_{\text{sea water}}$) = 1.025 gcm^{-3}

According to the law of floatation

$$\frac{v}{V} = \frac{\rho_{ice}}{\rho_{seawater}}$$

$$\text{or } \frac{v}{V} = \frac{0.92}{1.025} = 0.8976$$

$$\text{Or } v = (0.8976)V$$

$$\therefore \text{volume of iceberg above water} = 800 \text{ cm}^3 = V - 0.8976 V$$

$$\text{Or } v(1 - 0.8976) = 800$$

$$\text{Or } v = \frac{800}{(1 - 0.8976)}$$

$$\text{Or } v = 7812.5 \text{ cm}^3$$

9. A weather forecasting plastic balloon of volume 15 m^3 contains hydrogen of density 0.09 kg m^{-3} . The volume of an equipment carried by the balloon is negligible compared to its own volume. The mass of an empty balloon alone is 7.15 kg . The balloon is floating in air of density 1.3 kg m^{-3} .

Calculate: (i) The mass of hydrogen in the balloon, (ii) The mass of hydrogen and balloon, (iii) The total mass of hydrogen, balloon and equipment if the mass of equipment is $x \text{ kg}$, (iv) The mass of air displaced by balloon and (v) The mass of equipment using the law of floatation.

Solution

$$\text{Volume of plastic balloon} = 15 \text{ m}^3$$

$$\text{Mass of empty balloon} = 7.15 \text{ kg}$$

$$\text{Density of hydrogen} = 0.09 \text{ kg m}^{-3}$$

$$\text{Density of air} = 1.3 \text{ kg m}^{-3}$$

(i) mass of hydrogen in the balloon = volume of balloon \times density of hydrogen

$$\text{Mass of hydrogen in the balloon} = (15 \times 0.09) \text{ kg} = 1.35 \text{ kg}$$

(ii) mass of hydrogen and balloon = mass of empty balloon + mass of hydrogen in the balloon

Mass of hydrogen balloon = $[7.15 + 1.35] \text{ kg} = 8.5 \text{ kg}$

(iii) given mass of equipment = x

Total mass of hydrogen balloon and equipment = $(8.5 + x) \text{ kg}$

(iv) weight of air displaced by the balloon = upthrust = volume of balloon \times density of air $\times g$

Mass of air displaced = volume of balloon \times density of air
 $= 15 \times 1.3 = 19.5 \text{ kg}$

(v) using the law of floatation

Mass of air displaced = total mass of hydrogen , balloon and equipment

Or $19.5 = 8.5 + x$

Or $x = 11 \text{ kg}$

Thus mass of the equipment is 11 kg .